
# PROFESSIONALIZING PRIMARY SCHOOL MATHEMATICS TEACHER EDUCATORS

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## ABSTRACT

*Mathematics teacher educators in primary teacher education need expert knowledge and skills in teaching in primary school, in subject matter and research. Most starting mathematics teacher educators possess only part of this knowledge and skills. A professional development trajectory for this group is developed and tested, where a design based research is used to evaluate the design. This paper describes the professional development trajectory and it's design. We conclude that the professional development design should focus on mathematical knowledge for teaching, should refer to both teacher education and primary education, should offer opportunities for cooperative learning, and need to use practice based research as a developmental tool.*


## INTRODUCTION

Mathematics teacher educators in primary teacher education need high level mathematical content knowledge, deep theoretical knowledge on mathematics teaching in primary education, knowledge and skills in supporting student teachers in teacher education, and knowledge of teaching practice in primary education (Goffree & Dolk, 1995). Only few starting mathematics teacher educators combine all these competences.

In the Netherlands, starting mathematics teacher educators have diverse backgrounds and differently developed competencies. There is no professional standard or certification to become a mathematics teacher educator. Almost all starting mathematics teacher educators feel that they have to develop extra skills and knowledge. But what skills and knowledge those are varies depending on the educator in question.

Of course, new employees are rarely fully prepared for their new job or task. However, we notice that the support from colleagues for starting mathematics teacher educators on the job is mostly of practical nature. They get information on many relevant aspects of teacher education, such as the institute's curriculum, the digital system, the assessment processes and many more, but not on domain specific knowledge and skills that are necessary for providing high quality mathematics teacher education. From talks with starting colleagues at conferences and other gatherings a clear need came to the fore for more profound, theoretical, and specific professionalization trajectory. That is why we decided to develop and perform a professional development trajectory for starting mathematics teacher educators.

In this paper we consider the design of the professional development trajectory as design research (Bakker, 2018). We will sketch the process leading to formulating design principles. From these design principles we developed a design used in the trajectory. Then we elaborate on experiences in the professional development trajectory by providing an example from a morning session. This brings us to a critical analysis of the design and ideas for further development.

We thus answer the following research question:

What are characteristics of a professional development trajectory for starting mathematics teacher educators?


## METHOD

### Participants

Mathematics teacher educators were approached through advertisements in professional journals, on websites, and by notices at conferences. On the institutional level all Dutch teacher education institutes were approached.

The first cohort of 14 teacher educators commenced the professional development trajectory in 2020. In 2021, in the second cohort 13 teacher educators participated and in 2022, in the third cohort, 20 teacher educators. With these three cohorts teacher educators from 17 different teacher education institutes from all over the Netherlands participated. Several participants had to travel over two hours to get to the course location.

Participants' background, knowledge, and educational experience differed significantly. Some just started as teacher educators, whereas others had one or several years' experience as mathematics teacher educator. There were participants with experience as primary school teacher or math specialist in primary education, others had been mathematics teachers at secondary school. Some participants have a PhD in mathematics education, with extensive research experience and some teaching experience at university. Several participants had been educational advisor or textbook writer and developer for mathematics education. As a consequence there are large differences between participants in terms of subject matter knowledge, pedagogical content knowledge, teaching experience in primary education or teacher education, and experience in doing research.

### Towards design principles for the professional development trajectory

We developed a professional development trajectory taking into account the diverse population of relatively new mathematics teacher educators and the diverse skills and knowledge set they are supposed to develop. The skills and knowledge set consists of at least high level mathematical content knowledge (Oonk, Van Zanten,


& Keijzer, 2007), mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008), deep theoretical knowledge on mathematics teaching in primary education, knowledge and skills about supporting student teachers in teacher education (Oonk, 2009), and knowledge of teaching practice in primary education. To develop this, we estimated that educators would need support over an extensive period of time, we would have to know their starting level on the different aspects very well and taking good account of the different starting levels between them in order to optimally cater the sessions to their needs, and we would have to link the activities in the trajectory to primary school practice, teacher education practice, and insights from mathematics education literature. In several design sessions we devised relevant topics to include in the trajectory and the design principles we would abide by in designing the meetings in the trajectory and the activities therein. Finally, we characterized the professional development trajectory by the following features.

The trajectory:

1. has mathematical knowledge for teaching as a content and pedagogical framework,
2. consists of activities that are embedded in both mathematics teacher education practice and primary education practice,
3. offers opportunities for cooperative learning, and
4. uses practice based research as a developmental tool.

The design team consisted of five experienced mathematics teacher educators who work at three different teacher education institutes in the Netherlands. Their expertise is in practice-oriented research, curriculum development and professional development related to primary mathematics education and teacher education.

## General characteristics of the professional development trajectory

The professional development trajectory takes two years, with five meetings of six hours during a year. For each of these ten meetings the participants prepare specific tasks both individually and in small teams. These tasks generally involve some reading of literature and practical assignments in their teacher education, activities which they develop, carry out, and evaluate. The second year in the trajectory focuses on practice base research with associated assignments. Estimated time need for preparatory work for each meeting amounts to a maximum of 10 hours. The study load thus amounts to 160 hours over the two year period. Participants are facilitated by their institutions to be able to make this time investment.

Design team members also taught the course. Generally two course leaders are responsible for a meeting day, but the other course leaders strive to be actively present in these meetings as well. Thus ensuring the continuous exchange between experts through co-teaching and further development of the trajectory.

## Using the design principles to develop the professional development trajectory

### *1. The professional development trajectory has mathematical knowledge for teaching as a content and pedagogical framework*

The professional development trajectory is called 'verdiepingscursus' in Dutch, which means 'deepening course'. This implies that our participants are provided with more than practical lesson ideas and teacher educator skills, they are challenged to really develop their mathematical knowledge for teaching, both in subject matter knowledge as pedagogical content knowledge (Ball, Thames, & Phelps, 2008). The starting point and thus the steepness and length of their needed learning curve differs between the participants. We strive to challenge each participant on their own level (or slightly above) to make the connection between their mathematics teacher education practice and their mathematical knowledge for teaching through studying, reflecting, experimenting, and analyzing, thus allowing for growth in both domains. In the meetings we included several aspects of mathematical knowledge for teaching, sometimes focusing more on primary mathematics education, on mathematics teacher education, on (specialized) content knowledge, or on a combination of these. The meetings always are focused on one or two themes, to which the preparatory work is connected. The main themes of the meetings are:

1. The infrastructure of mathematics education in the Netherlands and abroad. In this we look into the extant journals, professional and scientific, conferences, associations, research institutes, funding agencies and much more.
2. The goals of primary mathematics education and the extent to which these are reached in the Netherlands and abroad.
3. The vision on primary mathematics education and mathematics teacher education, connected to the used textbooks.


4. Teaching and learning trajectories for primary mathematics education and mathematics teacher education.
5. Analyzing primary school students' and preservice teachers' solution strategies to be able to connect to their ways and levels of thinking.
6. Differentiation: from dealing with differences to using differences between students in primary education and teacher education
7. Developing a mathematical attitude of primary school students, preservice teachers, and mathematics teacher educators.
8. Developing higher-order thinking skills, like problem solving, and the skills teachers need to develop these in primary school and teacher education.
9. Developing mathematical literacy (functional numeracy) in primary school students and preservice teachers.
10. Developing design research skills to perform practice-oriented research in primary mathematics teacher education.

*2. The professional development trajectory consists of activities that are embedded in both mathematics teacher education practice and primary education practice*

Participants in the professional development trajectory are expected to develop on three important aspects. Firstly, they develop their own mathematical knowledge for teaching. They are challenged to apply this knowledge in their own teacher education practice. For example by discussing the goals and didactics of mathematics teacher education with their colleagues. Secondly, participants also implement new teacher education activities with their preservice teachers. When discussing, designing, experimenting, and evaluating this mathematical knowledge for teaching in teacher education practice the question comes to the fore how preservice teachers can apply this knowledge in their practice schools while teaching mathematics. The three levels in the professional development trajectory refer to activities in which the teacher educator, their students, or primary school students are exchanging and continually developing together.

*3. The professional development trajectory offers opportunities for cooperative learning*

The professional development trajectory offers a wide variety of learning activities where knowledge and experience of the leading teacher educators and practical and scientific sources play an important role. Equally important are the experiences, knowledge, and competences of the participants. The course, purposefully, offers many tasks and activities that are aimed at exchanging and evaluating experiences,


and offer opportunities to learn from and with peers. Such activities are for example activities in which participants, individually or in small peer groups, interview their colleagues in mathematics teacher education, have their student-teachers solve non-routine mathematical problems, look for mathematics in the world and in media, analyze textbooks and syllabi used in their institution, evaluate assessments and student work, design lesson activities and trial these in teacher education, investigate the vision of their teacher education institute on mathematics education, illustrate the teaching and learning trajectories used by their teacher education institutes, read literature of their choosing as well as provided literature, and answer their own research questions in that way, and share their findings in the meetings with other teacher educators. Plenary and small-group discussions are connected with theory, or form the basis for a new assignment. This learning with and from each other is intertwined with instructions or reflections from the course leaders. By this approach to learning we strive to have the participants prepared for and involved in an existing professional network or as developed within the trajectory, whereon they can still rely after the completion of the professional development trajectory. To facilitate these connections between the participants during the meetings there is always space built-in for informal exchanges in the form of lunches or walking tours.

### *4. The course uses practice based research as a developmental tool*

Depth and progress in the learning process of the participants is aimed at by using an inquiry-based approach into questions or problems from their own teacher education practice. Participants select and study literature, design possible solutions, and trial these in their own institution, all the while gathering data to analyze and evaluate their designed intervention (Bakker, 2018). Such inquiries allow both for solutions to local problems from practice and the development of participants' mathematical knowledge for teaching. In the first year of the professional development trajectory relatively small and well-defined inquiry activities are undertaken. In the second year, participants perform a research project during the entire year, in which they focus on investigating and designing an educational intervention in their own teacher education institute. The participants are grouped thematically and can thus divide the work and perform several cycles in their design research with different student groups and context. In addition to the thematic focus of the meetings of the trajectory, about half of the time in the meetings in the second year is dedicated to this research project.


## EXAMPLE MORNING SESSION: MATHEMATICAL ATTITUDE

To illustrate our approach, we will now describe some activities performed during a morning session in year 1. During this session, the central theme is the development of a mathematical attitude. Participants realize and experience that this ongoing development takes place among themselves, student teachers, and primary school students, and they will investigate how to encourage this.

In preparation for the meeting, the students have read Oonk and De Goeij's (2006) article on a mathematical attitude and prepare questions of the most appropriate way to work on the development of a mathematical attitude in primary school, and whether this follows or precedes basic skill development. One of the authors, Erica de Goeij, is present and lectures about the different aspects of a mathematical attitude. In addition to affective aspects, such as self-confidence and pleasure in mathematics, she also distinguishes reflective, inquisitive, critical, and communicative aspects of the mathematical attitude. This means that people with a mathematical attitude recognize mathematics in the world around them and explore how they can use mathematics to solve problems in everyday life. Verbalizing different approaches to solving problems and sharing and evaluating them with peers plays an important role in developing a mathematical attitude. Following the lecture, participants were given a meaningful task to solve: 'How many kilometres of toilet paper has been bought in the Netherlands last year?' Solving such a task requires knowledge of real-world measurements and quantities, and higher-order mathematical skills, such as critical and logical thinking and problem-solving skills. Participants first work on the question individually, then they discuss their problem approaches in small groups, and finally they have a plenary discussion and reflection.

One participant tries to work out the circumference of a toilet roll using the constant $\pi$ ($\approx 3.14$). She estimates the number of layers of paper and in this way approximates the length of toilet paper on one roll. While discussing this with some other participants, she discovered that this could not be correct, since not every layer on the roll has the same length. Others estimate the length of a sheet of toilet paper and the number of sheets used in the Netherlands over the past year. Estimates of the length of a sheet of toilet paper range from 10 to 15 centimetres. One participant found on the internet that 12 cm is the exact length of one sheet. A deviation of 2 or 3 cm seems not much, but on a number of about 62 billion sheets, it causes a big difference. It makes participants critically reflect on estimates and their impact.


Some participants do not use separate sheets in their problem approaches, but use the length of a roll of toilet paper. One participant knows that her living room measures 14 metres. She estimates that she can cover that distance about twice with a roll of toilet paper and derives that a roll of toilet paper has a length of about 30 metres. Most participants note their thoughts quite extensively and can explain their approach well. During the small-group discussions, quite a few calculation errors and diverse estimations come to light. One of the participants estimated the population of the Netherlands to be 15 million, while it is actually about 18 million at the moment (see Figure 1).

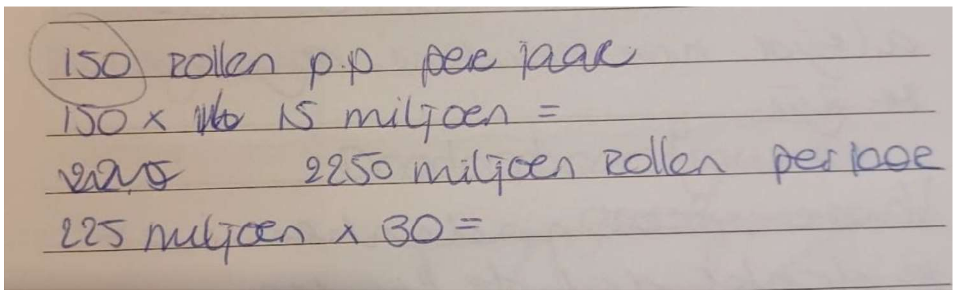


*Figure 1:* This participant estimates that one person uses 150 rolls of toilet paper a year and that there are 30 metres of paper on a roll. Multiplying the use per person by only 15 million inhabitants is too few and, she makes an error of a factor 10 in her calculation.

Another participant discovers that she should have multiplied her answer by the number of inhabitants. Yet another participant realises he should have multiplied the number of kilometres per day by 365 to arrive at the yearly use (see Figure 2). Things also went wrong occasionally when converting centimetres to kilometres. It becomes clear that some teacher educators struggle with such rich meaningful problems whereas others can solve these problems easily.

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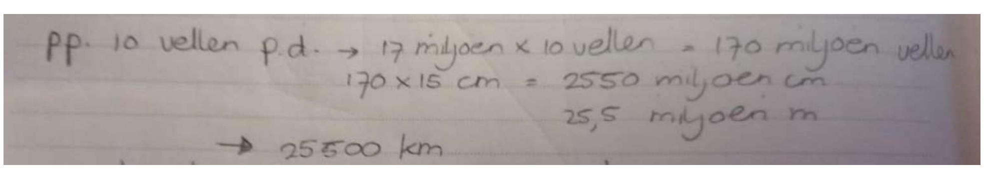


*Figure 2:* This participant estimates that 17 million people use 170 million sheets of toilet paper a day. She estimates the length of a sheet of toilet paper to be 15 cm. She forgets to multiply the distance found per day by the number of days per year.


During the plenary discussion, we noticed some participants being more eager to explain their own problem approach than questioning fellow participants. Trying to connect with the thinking and reasoning of the other problem solver is crucial in any learning process and should play a part at every level of education in order to achieve optimal development of mathematical thinking. Participants recognize this and agreed with the importance of this feature of teacher behavior. To make the participants more aware of the process of developing a mathematical attitude, we provide them with a list of characteristics of a mathematical attitude with the request to tick the characteristics that played a role during their individual work on the problem, and which were addressed during the small-group and plenary discussions. The participants believed that while working individually on the task, recognizing and applying mathematics in situations and being focused on appropriate numbers and on accuracy and completeness, were mostly present. While working together on the problem, being focused on alternative problem approaches, using mathematical language in collaboration with others, and being critical of the use of mathematics were more evident.

Participants also discovered how important the choice of an appropriate problem is in developing a mathematical attitude. The problem about the toilet paper was intriguing, required knowledge of the world, and appealed to higher-order thinking skills, but some participants considered the problem not urgent enough. They considered that it would be more valuable to use problems that you actually encounter in everyday life, for example, predicting the hight of your energy bill and how one’s behaviour may affect that amount to be paid.

In the second part of the morning, participants analyse the work of primary school students who worked on the same toilet paper problem. The students’ problem approaches also show many differences, especially in the ways they note their thinking. Participants experienced that it places high demands on the teacher to understand how students reason in rich meaningful problems, and especially to then conduct a classroom discussion in which students are learning from and with each other and thus developing their mathematical attitude. You need a lot of mathematical and didactic skills and knowledge as a teacher. Getting students to give respectful feedback on each other's work requires a safe classroom climate in which students are used to listening to and respecting each other. This, then, also requires strong pedagogical skills.

Finally, several participants decide to apply the toilet paper problem, the primary school student materials and the article, to a lesson on mathematical attitude in their


own teacher education program as well. The several aspects of the activities made them think about the development of mathematical attitude, and they hope to trigger this thinking process in their own student teachers as well. Perhaps this will subsequently encourage their student teachers to experiment with the problem in primary school practice.

## Reflection on the morning session

In the description of this example, the four design criteria are clearly recognizable. The content theme is mathematical attitude, the participants can grow in their mathematical knowledge for teaching by reading the article and by the input of guest speaker De Goeij. They also further construct their own knowledge by sharing and reflecting on experiences during the meeting. The three levels - teacher educators, preservice teachers, and primary school students - are continuously interchanging or simultaneously in the spotlight. While working on the toilet paper problem, participants become aware of characteristics of mathematical attitude needed for educators. They are challenged to think about the required teacher skills to develop a mathematical attitude. What should their students teachers know and be able to do? Finally, they analysed the work of primary school students and how primary school students can develop their mathematical attitude. Learning from and with each other takes place during small group work and plenary discussion. Thus, participants work collectively on their own mathematics skills and their views on the aims of mathematics education. The fourth design principle - practice-based research as a developmental tool - comes into play at the many moments during the meeting when participants are invited, individually or together, to systematically explore, describe and evaluate their own and the primary school's practice, using the knowledge and experiences gained before and during the meeting. That this encouragement to research-based thinking about their own teacher education and the primary education effectively inspired the participants is evidenced by the fact that during the practice-oriented research in the second year of the course several participants chose to do a practical research project concerning the development of their students' mathematical attitude.

## CONCLUSION AND DISCUSSION

### What did we learn from these experiences - in a (more) broad sense?

Looking back at the professional development trajectory, we can conclude that the trajectory, as described in the design principles, has clearly contributed to the further professionalization of the participants. Concretely, the trajectory provided them with a more profound knowledge of the domain, by inspiring them with theorical insights and use these to investigate practical situations. Also by discussing with their teacher education colleagues and trying to get a grasp of their vision on mathematics education, and by doing practice-oriented research into a particular subject. Additionally the exchanges between teacher educators have clearly contributed to a broader perspective on education, educational settings, and the content. The mathematics teacher educators in the professional development trajectory became a professional learning community and formed a network, on which they can continue to rely after finishing the trajectory. Finally they have been learning from the diverse contexts of teacher education institutes in their practice-oriented research project in the second year.

As such we can conclude that the knowledge gains we aimed for and that we strived to obtain using the design criteria were indeed achieved. The participants also provided their feedback on this to us:

“I found it especially interesting to read articles from different mathematics education journals and discuss these with the peer group” (cf. design principle 1)

“Already in the first meeting it became clear that everyone’s background and level was taken into account. Differentiation. I really appreciated that.”

“I really found it of added value to exchange ideas and especially experiences. As I work in a small institute these exchanges with mathematics teacher educators are rare.” (cf. design principle 3)


Notwithstanding the second quote, a challenge for the trajectory remains to really cater to the needs of all participants. Different participants gave different feedback on the different themes dealt with in the trajectory. As described above, due to the differences in previous experiences and knowledge the participants' needs differed greatly. Although we strived to incorporate different foci and levels in each meeting, participants did sometimes feel that things went too slowly, or too quickly. All in all it is clear that participants and the course leaders were positive about the content of the professional development trajectory and the development of the participants' mathematical knowledge for teaching therein. The designed professional development trajectory with a content-specific focus clearly contributes to the professionalization of new mathematics teacher educators.

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